

DETERMINATION OF CAPILLARY CONDUCTANCE OF
UNSATURATED CAPILLARY-POROUS MATERIALS

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A method of determining the capillary conductance of unsaturated porous materials with a moisture content close to saturation is described, and some results are given.

The motion of a liquid in capillary-porous materials is of great practical interest. An exact analytical solution of the problem is extremely difficult owing to the complexity and randomness of the pore space geometry. Hence, the empirical Darcy law

$$j = -K \nabla \phi. \quad (1)$$

is usually used to predict the motion of a liquid in capillary-porous materials.

The capillary conductance K , which depends on the structure of the sample and the properties of the liquid, is determined experimentally.

It was shown in [1] that the Darcy law can also be applied to a flow in unsaturated media. In this case, however, the capillary conductance is a function of the moisture content.

We consider a porous sample of volume V lying on a grid in a chamber and impregnated with liquid to a moisture content U . Let the initial air pressure above the sample in the chamber be P_i . An increase in pressure by ΔP will cause the liquid to flow out of the sample. The flow will continue as long as there is excess pressure in the liquid, equal to the difference between the external and capillary pressure $P = P_f - P_{cap}$. The excess pressure will decrease as the sample dries out, since the capillary pressure increases with reduction in moisture content. This pressure difference is the moving force of the process. The potential function can then be written in the form $\phi = P/\rho g$. Combining equation (1) and the continuity equation, we obtain

$$\frac{\partial U}{\partial t} = \frac{1}{\rho g} \cdot \frac{\partial}{\partial z} \left[K(U) \frac{\partial P}{\partial z} \right]. \quad (2)$$

The boundary conditions are based on the following considerations. At the lower boundary the liquid pressure is always atmospheric

$$P(0, t) = 0. \quad (3a)$$

There is no flow through the upper boundary of the sample

$$\left. \frac{\partial P}{\partial z} \right|_{z=L} = 0. \quad (3b)$$

In most cases (2) is nonlinear and is difficult to solve analytically. We can, however, make certain assumptions [2] which linearize this equation and enable us to solve it. We assume that ΔP is small enough for K to be regarded as approximately constant during the process. This means that only a small change in moisture content is permissible. This assumption allows us to take K out from under the differentiation sign. In addition, the assumption that the moisture content is a linear function of the capillary pressure gives

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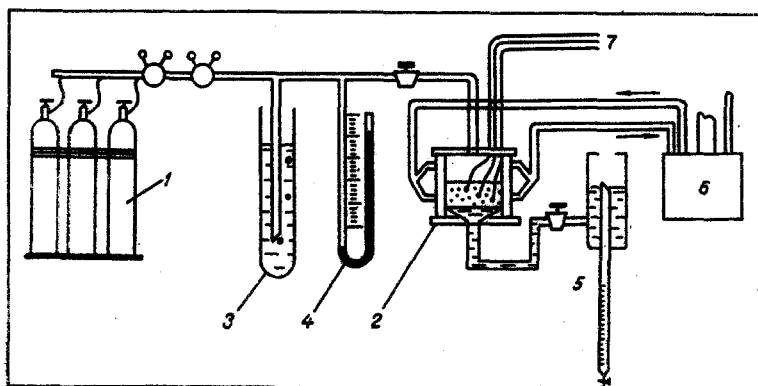


Fig. 1. Diagram of experimental apparatus: 1) air cylinders; 2) working chamber; 3) vessel for maintaining constant pressure; 4) manometer; 5) measuring buret; 6) thermostat; 7) thermocouples.

$$U(P) = a + b(P_f - P_{cap}), \quad (4)$$

where a and b are constants. Although none of the assumptions is absolutely accurate any required accuracy can be obtained by making ΔP sufficiently small. Substituting (4) in (2) we obtain

$$\frac{\partial P}{\partial t} = \frac{K}{\rho g b} \cdot \frac{\partial^2 P}{\partial z^2}. \quad (5)$$

The coefficient in front of the second derivative is just the hydrodynamic diffusion coefficient D of the liquid:

$$D = \frac{K}{\rho g (dU/dP)}.$$

Equation (5) with boundary conditions (3a) and (3b) can be solved by the method of separation of variables. Using the complete solution of this equation $P(z, t)$ and condition (4) we can write an expression for the moisture content

$$U(z, t) = a + b \frac{4\Delta P}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi z}{2L}\right) \exp(-\alpha^2 Dt), \quad (6)$$

where

$$\alpha = (n\pi/2L), \quad n = 1, 3, 5, 7 \dots$$

We are interested in the total moisture content of the sample

$$U(t) = \int_0^L AU(z, t) dz = aV + \frac{8b\Delta PV}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp(-\alpha^2 Dt). \quad (7)$$

The outflow at any instant t will then be

$$Q(t) = U_1 - U(t) = Q_0 \left[1 - \frac{8}{\pi^2} \exp(-\alpha^2 Dt) \right]. \quad (8)$$

This expression is used to calculate the capillary conductance in each interval of moisture content.

The apparatus used to determine the relationship $K = f(U)$ reproduced the boundary conditions specified for the solution of equation (2) (Fig. 1). Gas from the cylinders 1 entered the working chamber 2, which was a clear plastic cylinder 9.5 cm in diameter and 15 cm high in a thermostated jacket. The vessel 3 provided for regulation and maintenance of a constant pressure of a few centimeters of water column in the working chamber. The outflow was measured with a buret 5, and the pressure was measured with a U-tube manometer. The porous material was a bed of quartz sand 8 cm high. The grain diameter was 0.25–0.35 mm. Theoretically the porosity of a system composed of identical spheres should not depend on the particle size. In natural materials and carefully graded sands, however, the porosity depends on the particle size and the mode of packing and increases with reduction of diameter. An index, though not

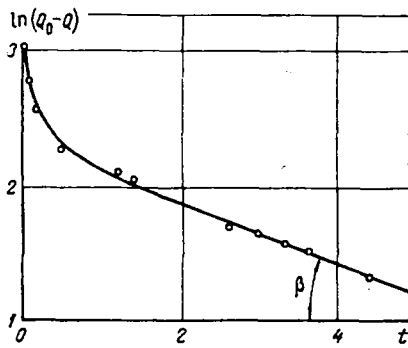


Fig. 2. Relationship $\ln(Q_0 - Q) = f(t)$ for quartz sand ($d \sim 0.25-0.35$ mm); $P_i = 34$ cm H_2O ; $P_f = 38$ cm H_2O ; $U_i = 29.8\%$; $U_f = 26.1\%$; $K = 110 \cdot 10^{-5}$ cm/min; t , min.

a measure of the capillary conductance is the active porosity – the ratio of the volume of all the communicating pores to the total volume of the sample. In the case of finely granular materials the active and absolute porosities are equal, since all the pores are available for movement of liquid. In the present investigation the porosity was measured by the pycnometric method [4] and was 39%. The temperature in the bed was monitored with thermocouples. As soon as the temperature in the bed had become steady the pressure was increased by ΔP and the outflow Q was measured as a function of time. When equilibrium corresponding to moisture content U was established the pressure was again increased. The pressure change ΔP was made small enough to ensure an insignificant change in moisture content and the fulfilment of the condition $K \approx \text{const}$. From the experimental data we constructed graphs of the relationship $\ln[Q_0 - Q(t)] = f(t)$, one of which is shown in Fig. 2. As Fig. 2 shows, the graph is a straight line with angle of inclination β . The initial portion of

the curve was not used for the calculation, since in the case of small times not only the first term, but also terms of higher order, in the expansion in the series in (8) have to be included. Proceeding from equation (8) we can determine D from the gradient:

$$D\alpha^2 = \text{tg } \beta$$

and, using the determination of coefficient D and the total outflow Q_0 , we obtain a formula for the capillary conductance

$$K = \frac{Q_0 \text{tg } \beta}{\Delta P} \cdot \frac{\rho g}{V\alpha^2}$$

Measurements of the capillary conductance were made for moisture contents 21.5–38%. Figure 3a shows the obtained relationship between K and the moisture content in a bed of quartz sand ($d \sim 0.25-0.35$ mm). It is difficult to compare these values of K with previously published values. Most of the published values of capillary conductance are for unsaturated soils and ground. In addition, these published values relate to a region of moisture contents far from saturation and, hence, K decreases steadily with reduction of U .

As Fig. 3a shows, the curve $K = f(U)$ has two maxima in a particular region at high moisture contents. To facilitate the explanation we will use Fig. 3b, which shows a plot of capillary conductance against external pressure. The outflow from the bed begins when $P_i = 19$ cm H_2O . The first maximum is observed at $P_2 = 24$ cm H_2O and the second at $P_3 = 31$ cm H_2O . Since at equilibrium the external pressure is balanced by the capillary pressure, we can link pore radii r_1 , r_2 , and r_3 with pressures P_1 , P_2 , and P_3 . The calculation showed that $r_2 = 0.0060$ cm, and $r_3 = 0.0046$ cm. A sand bed is a system penetrated by capillary tubes with their axes following the line of the surface of the spherical particles. The capillary radii alternate regularly from the maximum to the minimum value. In the case of hexagonal packing the pores are

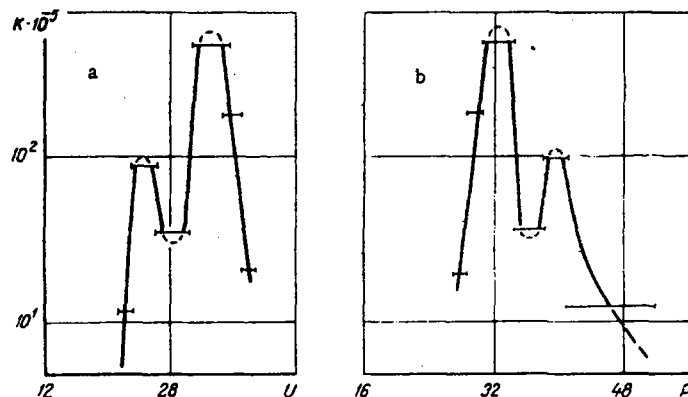


Fig. 3. Capillary conductance of quartz sand as a function of: a) moisture content; b) external pressure. K , cm/min; P , cm H_2O ; U , %.

of two types – tetrahedral ($R_{\text{tetr}} = 0.288 R_{\text{part}}$) and rhombohedral ($R_{\text{rhom}} = 0.414 R_{\text{part}}$). The minimum pore radius is $0.155 R_{\text{part}}$ [3]. In a real porous medium, however, where the particle radii have a definite spread, there are also pores with other radii, but these are few in number. When the pressure is increased water begins to flow out of the largest pores. As the pressure increases further the sample becomes drier. When the external pressure is equal to the capillary pressure in the rhombohedral pore (in this case $P_{\text{rhom}} = 23 \text{ cm H}_2\text{O}$, $R_{\text{rhom}} = 0.0062 \text{ cm}$), the capillary conductance increases and there is a maximum on the curve of $K = f(U)$. This is in good agreement with experiment, since the values of P_{rhom} and R_{rhom} are close to $P_2 = 24 \text{ cm H}_2\text{O}$ and $r_2 = 0.0060 \text{ cm}$.

The second maximum of capillary conductance corresponds to the outflow from tetrahedral pores ($R_{\text{tetr}} = 0.0043 \text{ cm}$; $P_{\text{tetr}} = 33 \text{ cm H}_2\text{O}$), which is also close to the experimental values $P = 31 \text{ cm H}_2\text{O}$ and $r_3 = 0.0046 \text{ cm}$. Since there are only half as many tetrahedral pores as rhombohedral pores and the capillaries of smaller radius offer greater resistance to motion of the liquid, this second maximum will be smaller than the first.

Thus, the capillary conductance of any bulk porous material can be measured. The behavior of the curves of K and moisture content against pressure provide information about the pore radius distribution.

NOTATION

j	is the bulk liquid flow;
K	is the capillary conductance;
U	is the moisture content;
Φ	is the potential function;
P_i	is the initial pressure;
P_f	is the final pressure;
L	is the bed height;
V	is the bed volume;
ρ	is the density of liquid;
g	is the acceleration due to gravity;
D	is the hydrodynamic diffusion coefficient;
Q	is the outflow;
t	is the time;
r, R	are the pore radii;
R_{part}	is the particle radius.

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